Feb 11

1. Project — Proposal: Feb 28
— Presentations: April 13 & 15 — in class
— Reports: April 15

Goal: * demonstrate understanding
* getting around in the literature
aim for a "little" review
focus: clarity

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2. Last time: $\underbrace{\forall \pi: q^\pi \in_{\varepsilon_{apx}} \mathcal{F}}_{(B2)_{\varepsilon_{apx}}}$ + opt. design A.P.I. $\Rightarrow \delta = \frac{1}{(1-\gamma)^2}\sqrt{d}\,\varepsilon_{apx}$ feasible.

$\exp\left(\frac{d}{8}\left(\frac{\varepsilon_{apx}}{\delta}\right)^2\right)$

Today: $\varepsilon_{apx} = 0$, but relax $(B2)_\varepsilon$!

Low < query / compute > cost local planners?

$\operatorname{argmax}_a q^*(a) = \operatorname{argmax}_a \varphi(a)^T\theta_*$ $\quad \delta = 1/2$

2.1 $\quad q^* \in \mathcal{F}$, H-horizon $\quad \#q = e^{\Omega(H \wedge d)}$

$\boxed{A = 2^d}$

2.2 $\quad v^* \in \mathcal{F}$ $\quad$ ⊛ infinite hor, $A = \Theta(d)$

$\varphi: S \to \mathbb{R}^d$ $\quad \#q = e^{\tilde{\Omega}(d)}$

⊛ H-horizon, $A = O(1)$

$v^*(s) = \varphi(s)^T\theta_*$

$\#q = \text{poly}\left(\left(\frac{dH}{\delta}\right)^A, B\right)$

$\|\theta_*\|_2 \le B \qquad \|\varphi\| \le 1$

$\boxed{q^* \in \mathcal{F}}$

H-horizon problem

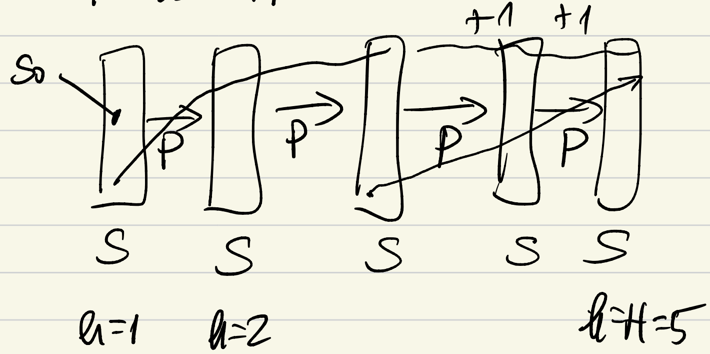
d : $\quad \varphi_h : S \times A \to \mathbb{R}^d$

$v^\pi = (v_h^\pi)$ $\quad \pi = (\pi_1, \dots, \pi_H)$

$v_h^\pi : S \to \mathbb{R}$

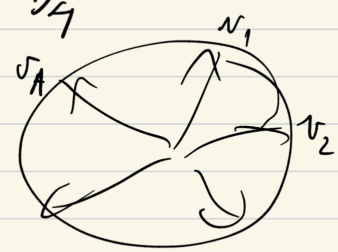
$1 \le h \le H$

$$\boxed{\begin{array}{l} q_h^*(s,a) = \varphi_h(s,a)^T \theta_* \\ \exists \theta_* \in \mathbb{R}^d \qquad h \in [H] \end{array}}$$



$A \approx e^d \quad \Rightarrow$ J-L Lemma

$\exists \; v_1, \dots, v_A \in S_2^d \qquad |\langle v_a, v_b \rangle| \le 1/4$



$q_h^*(s,a) = r_a(s) + \underbrace{q_{h+1}^*(s_a, a^*)}$

$a = a^*$

$1 = r_{a^*}(s) + 1$

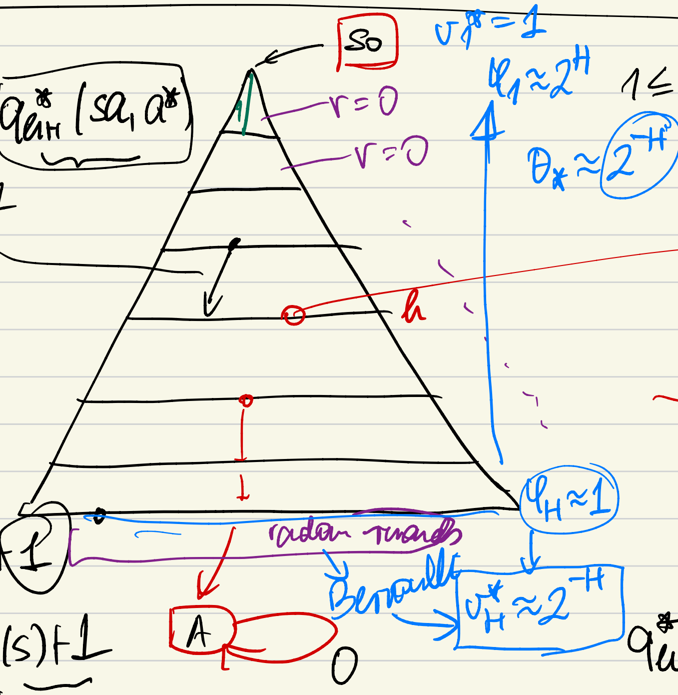
$\overset{\parallel}{0}$ $\quad r_a(s)$

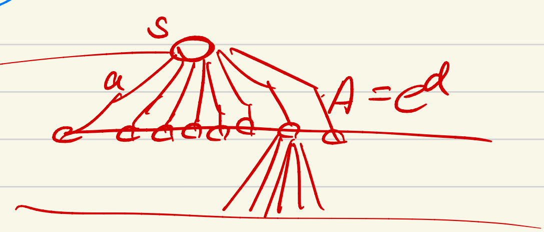
$a \neq a^*$

$-\frac{1}{4} \le q_h^*(s,a) \le \frac{1}{4}$

$[-\frac{1}{4}, \frac{1}{4}] = r_a(s) + 1$

$\underbrace{\varphi_h(s,a)^T \theta_*}_{\overset{\parallel}{1}} = \underbrace{r_a(s)}_{\overset{\parallel}{0}} + 1$



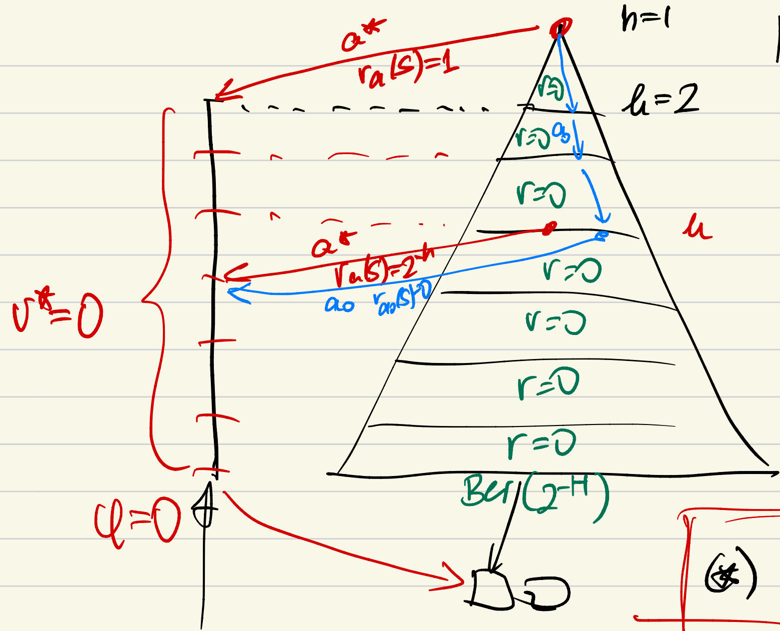


$\varphi_h(s,a) = v_a \;;\; \theta^* = v_{a^*}$

$q_h^*(s,a) = \langle v_a, v_{a^*} \rangle = \begin{cases} 1, & a = a^* \\ \le \frac{1}{4}, & a \neq a^* \end{cases}$

$\varphi_h(s,a) = \underbrace{\sigma_{sa}}_{\uparrow} v_a$

$\varphi_h(s,a) = (c_a, \sigma_{sa} v_a) \in \mathbb{R}^{dH}$



$\|\varphi_1\| \approx 2^H \qquad v_1^* \simeq 1 \qquad \|\theta_1^*\| \simeq 2^{-H}$

$$\varphi_h(s,a) = (c_h, \sigma_{sa} v_a)$$

$$\theta^* = c \cdot v_{a^*}$$

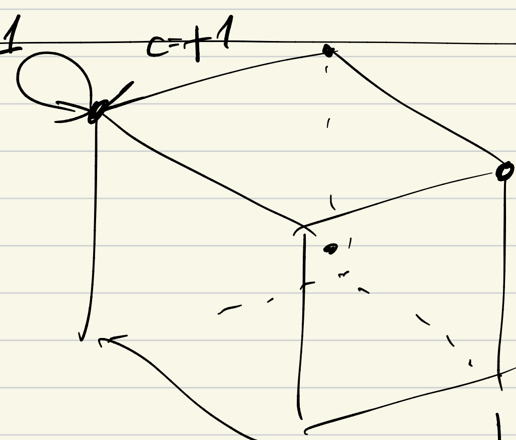
2^{-H}

$\|\varphi_H\| \approx 1 \qquad v_H^* \approx 2^{-H}$

$$\boxed{(*) \quad q_h^*(s,a) = \varphi_h(s,a)^T \theta^*} \qquad \forall (s,a)$$

$(\exists\, c_h, \sigma_{sa}, c)$ s.t. $(*)$ holds.

Infinite-horizon



$\boxed{S = \{-1,0,1\}^d}$

$\boxed{2^d = |\{-1,1\}^d|}$

$s \in \{-1,0,1\}^d$

$d - s^T(s^*)$

$\varphi(s) = (1, s)$

$$\boxed{v^*(s) = \varphi(s)^T (d, -s^*)}$$